THERMAL CALCULATION OF COMBINED TRANSPIRATION – LIQUID COOLING OF DISCHARGE-CHAMBER WALLS OF AN ELECTRIC-ARC HEATER

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UDC 536.12:533.9.15

The article presents a theoretical calculation of the temperature profiles and optimal thicknesses of porous and solid cooled walls, which provide a steady-state operation of an electric-arc heater with transpiration compression of the arc and combined cooling of the discharge-chamber walls.

To increase the parameters of a low-temperature plasma jet (temperature, enthalpy) produced by means of electric-arc heaters, various methods of stabilization (compression) of the electric arc are used.

The idea of stabilization consists in creating conditions under which the volumetric heat-release rate in the discharge chamber would be maximum. A serious difficulty arising in this case is the lack of materials able to withstand the enormous heat loads which the walls of the discharge chamber experience. The use of forced water cooling of the discharge chambers permits abstracting heat fluxes up to about 10 kW /cm^2, which imposes limitations on the possibilities of electric-arc heaters.

The method of transpiration cooling of the discharge chamber wall of the heater by blowing a coolant through pores in the wall makes it possible to eliminate the limitation indicated above, but to provide reliable heat protection of the discharge-chamber walls, the flow rate of the working gas required is such that the resultant mean-mass parameters (temperature, enthalpy) of the escaping jet are about the same as with water cooling.

Here we will consider the problem of not purely transpiration cooling but combined transpiration –liquid cooling in which part of the heat flux will be returned to the main flux by the injected working gas and part will be abstracted by forced water cooling. The use of this method presupposes a simultaneous improvement of the conditions of arc stabilization and cooling of the discharge-chamber walls of the heater. In comparison with purely transpiration cooling, the required flow rate of the working gas will be smaller, which will increase the output parameters of the jet.

The mathematical statement of the problem consists in an investigation of heat transfer in a bilayer cylindrical wall, one layer of which is porous and the second is solid (Fig. 1). The heat flux is delivered to the inside surface of the wall and the outside surface is bathed by the cooling medium. Channels conducting the working gas to the porous layer are made in the solid layer at the site of contact.

It is necessary to obtain the expression for determining the optimal thicknesses of the layers and expression for the temperature profiles in both layers and to calculate the external heat transfer accomplished by liquid cooling.

Calculation of the Porous Insert. We will consider the steady-state, one-dimensional case of heat transfer in a porous cylindrical wall during movement of the coolant in the pores of the wall. The equation of thermal conductivity in the given case has the form [5]

$$r\frac{d^2t}{dr^2}+(1+\varepsilon)\frac{dt}{dr}=0, \tag{1}$$

where $\varepsilon = c_g j_{cg} r_0/\lambda_{ef}$ is a dimensionless parameter. In Eq. (1) the term with parameter ε represents the change of enthalpy of the cooling gas. The effective thermal conductivity λ_{ef} for metallic porous materials

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 3, pp. 512-516, March, 1970. Original article submitted June 2, 1969.

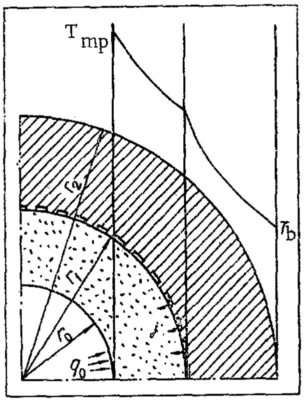

Fig. 1. Bilayer porous diaphragm (cross section).

manufactured by sintering for a known value of the volume portion of voids V can be determined by the formula [1]

$$\lambda_{ef} = \frac{\lambda_m}{0{,}242} \; \frac{1-(V+0.523)^{1/3}}{(V+0.523)^{1/3}} \, . \tag{2}$$

The solution of Eq. (1) for boundary conditions

$$t\,|_{r=r_1} = t_1, \quad -\lambda_{ef}\,\frac{dt}{dr}\bigg|_{r=r_0} = q_0$$

has the form

$$t = t_1 + \frac{q_0 r_0}{\lambda_{ef}\varepsilon}\left[\left(\frac{r_0}{r}\right)^{\varepsilon} - \left(\frac{r_0}{r_1}\right)^{\varepsilon}\right] \, . \tag{3}$$

The thickness of the porous insert $h = r_1 - r_0$, providing steady-state operation, can be found from the condition $t(r_0) = t_2$, where $t_2 < t_{mp}$; t_{mp} is the melting point of the material of the insert (for example, the melting point for porous metal). Denoting $t_2 - t_1 = \Delta t$, we find from Eq. (3)

$$r_1 = \frac{r_0}{\sqrt[\varepsilon]{1 - \Delta t\, c_g j_{cg}/q_0}} \tag{4}$$

or

$$h = r_0\,\frac{1 - \sqrt[\varepsilon]{1 - \Delta t c_g j_{cg}/q_0}}{\sqrt[\varepsilon]{1 - \Delta t c_g j_{cg}/q_0}} \, . \tag{5}$$

In the case when $\Delta t_g j_{cg}/q_0 \ll 1$, which will occur for certain operating conditions of the electric-arc heater, we can obtain simpler formulas by expanding (4) and (5) in series:

$$r_1 = r_0 + \frac{\Delta t \lambda_{ef}}{q_0};$$

$$h = \frac{\Delta t \lambda_{ef}}{q_0} \, .$$

Calculation of Temperature Profiles in Discharge-Chamber Wall and Thickness of the Shell of the Water-Cooling System. We will consider the simultaneous solution of the following two problems (see Fig. 1):

a) the problem of cooling a porous cylinder in the presence of the injection of the working gas:

$$r\,\frac{d^2 t_1}{dr^2} + (1+\varepsilon)\,\frac{dt_1}{dr} = 0, \quad r_0 \leqslant r \leqslant r_1,$$
$$t_1\,|_{r=r_0} = T_{mp}\,; \tag{6}$$

b) the problem of heat transfer in a cylindrical wall (solid) on the inside surface of which are made channels for conducting the working gas to the porous layer:

$$\frac{d}{dr}\left(r\,\frac{dt_2}{dr}\right) = 0, \quad r_1 < r < r_2,$$
$$t_2\,|_{r=r_2} = T_b \, . \tag{7}$$

The conditions of coupling the problems on the boundary $r = r_1$ are as follows:

$$t_1\,|_{r=r_1} = t_2\,|_{r=r_1},$$
$$\lambda_{ef}\,\frac{dt_1}{dr}\bigg|_{r=r_1} = \lambda_{ef}\,\frac{dt_2}{dr}\bigg|_{r=r_1} . \tag{8}$$

The equivalent thermal conductivity λ_{eq} for the solid cooled wall is calculated with consideration of the area of its contact with the porous wall (with consideration of the thermal resistance of the contact [6]).

The solution of problems (6)-(7) with consideration of coupling condition (8) gives the following expressions for the temperature profile in the bilayer wall of a discharge chamber of an electric-arc heater:

$$t_1 = T_{\text{mp}} + \frac{\Delta T\,(r_0^{-\varepsilon} - r^{-\varepsilon})}{r_1^{-\varepsilon}\left(\frac{\lambda_{\text{ef}}}{\lambda_{\text{eq}}}\ln\frac{r_1}{r_2} - \left(\frac{r_1}{r_0}\right)^{\varepsilon} - 1\right)}; \tag{9}$$

$$t_2 = T_{\text{b}} + \frac{\lambda_{\text{ef}}}{\lambda_{\text{eq}}} \cdot \frac{\Delta T \ln\frac{r}{r_2}}{\frac{\lambda_{\text{ef}}}{\lambda_{\text{eq}}}\ln\frac{r_1}{r_2} - \left(\frac{r_2}{r_1}\right)^{\varepsilon} - 1}. \tag{10}$$

The formula for the thickness $h_1 = r_2 - r_1$ of the cooled wall can be obtained from Eq. (9) with the use of condition:

$$-\lambda_{\text{ef}} \frac{dt_1}{dr}\bigg|_{r=r_0} = q_0.$$

After uncomplicated mathematical transformations we obtain the expression

$$h_1 = r_1\left[\exp\left\{-\left[\left(\frac{\lambda_{\text{ef}}\Delta T}{q_0 r_1} - 1\right)\left(\frac{r_1}{r_0}\right)^{\varepsilon} - 1\right]\right\} - 1\right]. \tag{11}$$

Calculation of External Heat Transfer. External (liquid) cooling of the discharge chamber of the electric-arc heater is accomplished by pumping the cooling liquid through the slotted channel of the wall being cooled. To calculate external heat transfer we must determine the value of the heat-transfer coefficient for given values of the quantities determining the conditions of heat transfer and calculate the flow velocity of the liquid in the cavity of the cooling jacket and required flow rate of the cooling medium.

If the inside surface of the discharge chamber absorbs the heat flux q_0, the flux

$$q_{\text{st}} = q_0\left(\frac{r_0}{r_1}\right)^{\varepsilon+1}\left(\frac{r_1}{r_2}\right)^2. \tag{12}$$

will penetrate to the water-cooled surface (wall). This expression can be obtained easily by using only the expressions for the temperature profiles in the wall (9) and (10) and the condition

$$-\lambda \frac{dt}{dr}\bigg|_{r=r_0} = q_0.$$

The value of the heat-transfer coefficient α needed for providing abstraction of the heat flux is determined from Newton's law:

$$\alpha = \frac{q_{\text{st}}}{(t_f - t_w)}. \tag{13}$$

The temperature of the cooled surface, t_f, should not exceed the boiling point of the cooling liquid at a known pressure in the system. Otherwise the phenomenon of "film" boiling will occur in the system, markedly affecting adversely the conditions of heat transfer and resulting in local overheating of the wall and consequently in destruction.

To determine the flow velocity of the cooling fluid necessary for heat transfer, we use the empirical relation obtained in [2] for straight and smooth pipes:

$$\text{Nu}_f = 0.021\,\text{Re}_f^{0.80}\,\text{Pr}_f^{0.43}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25}. \tag{14}$$

When $l/d_{\text{eq}} < 50$ (d_{eq} is the equivalent diameter of the channel; l is its length) heat transfer occurs somewhat more intensely and therefore α must be multiplied by a correction factor ε_1 [3]. Further, to take into account the curvature of the pipe characteristically affecting an increase of the heat-transfer coefficient, it must be multiplied by a correction factor

$$\varepsilon_R = 1 + 1.77\frac{d_{\text{eq}}}{R},$$

where R is the radius of an annular channel. With consideration of all the aforementioned comments we can obtain the following formula for calculating the average velocity of the cooling fluid:

$$w_f = \left(\frac{\nu}{d}\right)_f \sqrt[0.80]{\frac{\alpha \varepsilon_R \varepsilon_l}{0.021\lambda}\left(\frac{d}{\nu}\right)^{0.43}\left(\frac{\left(\frac{\nu}{d}\right)_w}{\left(\frac{\nu}{d}\right)_f}\right)^{0.25}}. \tag{15}$$

The optimal flow rate of the cooling fluid through the cross section of the cavity S_1 is found from the formula:

$$G = \rho w_f S. \tag{16}$$

In this article the problem is solved on the assumption that c_g and λ_{ef} are constants. In reality heat capacity and thermal conductivity are functions of temperature, but for small temperature differences in a porous insert (of the order of 300-350°C in our case) the error caused by the given simplification does not exceed 7-8%, which is completely acceptable for heat-engineering calculations.

NOTATION

c_g	is the heat capacity of the cooling gas;
j_{cg}	is the specific flow rate of cooling gas through the inside surface of the diaphragm;
r_0	is the inside radius of the porous diaphragm;
λ_m	is the thermal conductivity of the diaphragm material;
r_1	is the outside radius of the porous diaphragm;
r_2	is the outside radius of the cylinder being cooled;
q_0	is the thermal flux to inside surface of the porous diaphragm;
ΔT	is the temperature difference on the bilayer wall;
h	is the thickness of the porous diaphragm;
T_b	is the boiling point of the cooling medium;
T_{mp}	is the melting point of the material of the porous diaphragm;
h_1	is the thickness of the water-cooled wall;
q_{st}	is the thermal flux to the water-cooled wall;
Re and Pr	are dimensionless numbers.

Subscripts

f and w	indicate the quantity pertains, respectively, to conditions in an undisturbed flow and on the surface of the body.

LITERATURE CITED

1. A. Misnar, Thermal Conductivity of Solids, Liquids, and Gases and Their Compositions [Russian translation], Moscow (1968).
2. S. S. Kutateladze and V. M. Borishanskii, Handbook on Heat Transfer [in Russian], Gosénergoizdat (1959).
3. M. A. Mikheev, Principles of Heat Transfer [in Russian], Gosénergoizdat (1958).
4. N. N. Rykalin, Thermal Principles of Welding [in Russian], Gosénergoizdat (1954).
5. A. V. Lykov and B. M. Smol'skii (editors), Heat and Mass Transfer in Capillary-Porous Bodies [in Russian], Nauka i Tekhnika, Minsk (1965).
6. Yu. P. Shlykhov and E. A. Ganin, Contact Heat Transfer [in Russian], Gosénergoizdat (1963).